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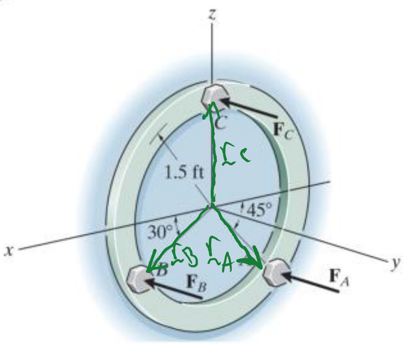



The three parallel bolting forces act on the circular plate, such that $F_A = F_B = F_C = F > 0$. Replace the force system by a resultant force $\boldsymbol{F}_R$ and a resultant moment $(\boldsymbol{M}_R)_O$ about the origin.

equal value

Find resultant force $F_R$:

$$F_R = \sum F = F_A + F_B + F_C$$

$$\boxed{F_R = -3F \cdot \hat{j}}$$

Position vectors:

$$r_A = r \cdot \left(-\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{k}\right) = -r \cdot \frac{\sqrt{2}}{2} \cdot (\hat{i} + \hat{k})$$

$$r_B = \frac{r}{2}\left(\sqrt{3}\,\hat{i} - \hat{k}\right)$$

$$r_C = r \cdot \hat{k}$$

Find resultant moment $M_r$:

$$M_R = \sum r \times F = r_A \times F_A + r_B \times F_B + r_C \times F_C$$

$$= r_A \times (-F \cdot \hat{j}) + r_B \times (-F \cdot \hat{j}) + r_C \times (-F \cdot \hat{j})$$

$$= (r_A + r_B + r_C) \times (-F \cdot \hat{j})$$

collect terms
Maintain cross-product order!

$$= -r \cdot F \cdot \left[\hat{i}\left(\underbrace{-\frac{\sqrt{2}}{2}}_{\text{from } r_A} + \underbrace{\frac{\sqrt{3}}{2}}_{\text{from } r_B}\right) + \hat{k} \cdot \left(\underbrace{-\frac{\sqrt{2}}{2}}_{\text{from } r_A} \underbrace{- \frac{1}{2}}_{\text{from } r_B} \underbrace{+ 1}_{\text{from } r_C}\right)\right] \times \hat{j}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

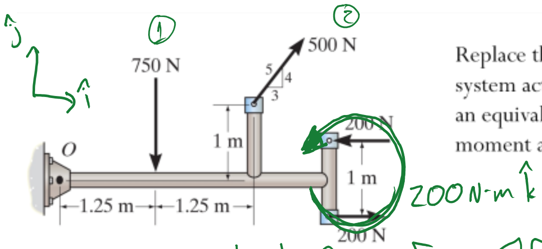
$$\hat{k} \times \hat{j} = -\hat{i}$$

$$= -r \cdot F \cdot \left[\left(\frac{\sqrt{3} - \sqrt{2}}{2}\right)\hat{i} + \left(\frac{1 - \sqrt{2}}{2}\right)\hat{k}\right] \times \hat{j}$$

$$= -r \cdot F \cdot \left[\left(\frac{\sqrt{3} - \sqrt{2}}{2}\right)\hat{k} + \left(\frac{1 - \sqrt{2}}{2}\right)(-\hat{i})\right]$$

$$M_R = r \cdot F \cdot \left[\left(\frac{1 - \sqrt{2}}{2}\right)\hat{i} + \left(\frac{\sqrt{2} - \sqrt{3}}{2}\right)\hat{k}\right]$$

$$\underset{\sim}{M}_R \approx r \cdot F \cdot \left( -0.207\,\hat{i} - 0.159\,\hat{k} \right)$$




Replace the force and couple system acting on the member by an equivalent force and couple moment acting at point O.

Find resultant force $\underset{\sim}{F}_R = \Sigma \underset{\sim}{F} = \overbrace{(-750 N \hat{j})}^{\underset{\sim}{F}_1} + \overbrace{(500 N)\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)}^{\underset{\sim}{F}_2}$

$$= (300 N)\hat{i} + (-750 + 400) N \cdot \hat{j}$$

$$F_R = (300 N \hat{i} - 350 N \hat{j})$$

$$(\underset{\sim}{M}_R)_O = \Sigma \underset{\sim}{r} \times F + \Sigma M$$ couple moments

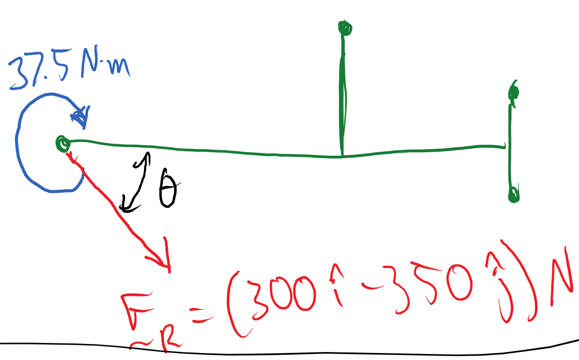
$$= \underset{\sim}{r}_1 \times \underset{\sim}{F}_1 + \underset{\sim}{r}_2 \times \underset{\sim}{F}_2 + \underset{\sim}{M}_c$$

$$= \underbrace{(1.25 m \hat{i}) \times (-750 N \hat{j})} + (2.50 m \hat{i} + 1 m \hat{j}) \times (300 N \hat{i} + 400 N \hat{j}) + (200 N \cdot m)\hat{k}$$

$$= -937.5 N \cdot m \hat{k} + 1000 N \cdot m \hat{k} - 300 N \cdot m \hat{k} + 200 N \cdot m \hat{k}$$

$$= (-937.5 + 900) N \cdot m \cdot \hat{k}$$

$$\boxed{(\underset{\sim}{M}_R)_O = -37.5 N \cdot m \cdot \hat{k}}$$




$$\theta = \tan^{-1}\left(\frac{350}{300}\right) = 49.4^\circ$$

$$\theta = \tan^{-1}\left(\frac{350}{300}\right) = 49.4°$$

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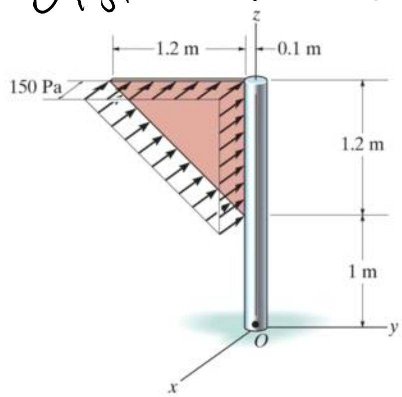
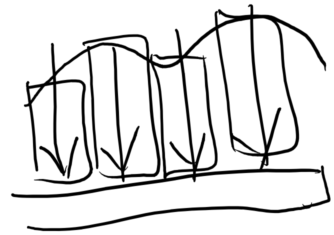
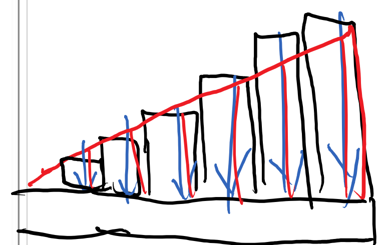
Distributed loads



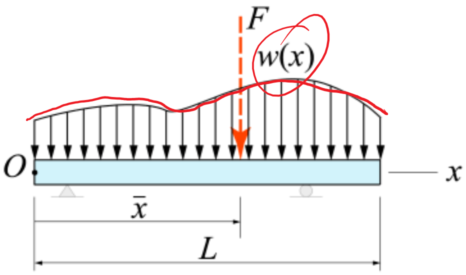





The lumber places a distributed load (due to the weight of the wood) on the beams. To analyze the load's effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

# Reduction to a simple distributed load




In structural analysis, we often are presented with a **distributed load** $w(x)$ (force/unit length) and we need to find the equivalent loading $F$.

Example of such forces are winds, fluids, or the weight of items on the body's surface.

dimensions of $w(x)$: $\frac{\text{force}}{\text{length}}$

Goal: convert $w(x)$ to a point force $F$ and find its point of application $\bar{x}$.

Require: $F = \int_0^L \underbrace{w(x)}_{\frac{\text{force}}{\text{length}}} \cdot \underbrace{dx}_{\text{length}} = \ldots \Big|_0^L$

remember to include units in the integral limits at the final step

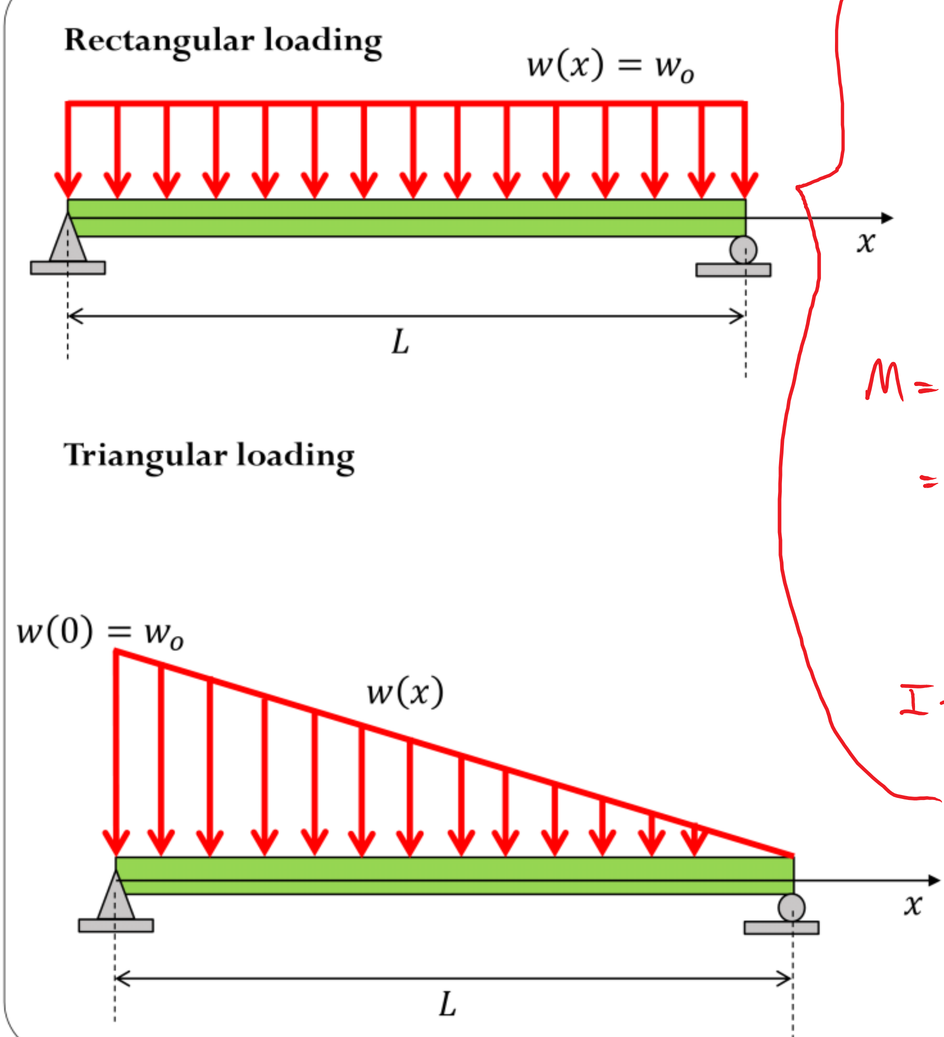
dimension of the limits always match the dimensions of $d(\ )$ — (here, dx)

and $M = \int_0^L w(x) \cdot x \cdot dx = \bar{x} \cdot F$

Combine both equations:

$$M = \bar{x} \cdot F \Rightarrow \underbrace{\bar{x}}_{\text{length}} = \frac{M}{F} = \frac{\int_0^L \overbrace{w(x)}^{\frac{\text{force}}{\text{length}}} \cdot \overbrace{x}^{\text{length}} \cdot \overbrace{dx}^{\text{length}}}{\int_0^L \underbrace{w(x)}_{\frac{\text{force}}{\text{length}}} \cdot \underbrace{dx}_{\text{length}}}$$

$\boxed{\bar{x} \text{ is the centroid of } w(x)}$




$$F = \int_0^L w(x) \cdot dx = \int_0^L w_o \cdot dx = w_o \cdot x \Big|_0^L = w_o \cdot L$$

$$\boxed{F = w_o \cdot L}$$

$$M = \int_0^L w_o \cdot x \cdot dx = w_o \int_0^L x \cdot dx = w_o \cdot \tfrac{1}{2} x^2 \Big|_0^L = \frac{w_o \cdot L^2}{2}$$

$$\boxed{M = \frac{w_o \cdot L^2}{2}}$$

If $M = \bar{x} \cdot F$, then $\bar{x} = \frac{M}{F} = \frac{\left(\frac{w_o \cdot L^2}{2}\right)}{w_o \cdot L}$

$\Rightarrow \boxed{\bar{x} = \frac{L}{2}}$

For a triangular loading:

A linear distribution: $w(x) = m \cdot x + b$

$w(0) = w_o \Rightarrow b = w_o$

$w(L) = 0$

Slope: $m = -\frac{w_o}{L}$

Thus, $w(x) = -\frac{w_o}{L} x + w_o$.

Rewrite as follows: $\underbrace{w(x)}_{\frac{\text{force}}{\text{length}}} = \underset{\frac{\text{force}}{\text{length}}}{w_0} \cdot \underbrace{\left(1 - \frac{x}{L}\right)}_{\text{dimensionless}}$

Check dimensions:

$$\boxed{F} = \int_0^L w_0 \cdot \left(1 - \frac{x}{L}\right) dx = w_0 \cdot \int_0^L \left(1 - \frac{x}{L}\right) \cdot dx$$

$$= w_0 \cdot \left(x - \frac{x^2}{2L}\right)\Big|_0^L = w_0 \cdot \left(L - \frac{L^2}{2L}\right) = \boxed{\frac{w_0 \cdot L}{2}}$$

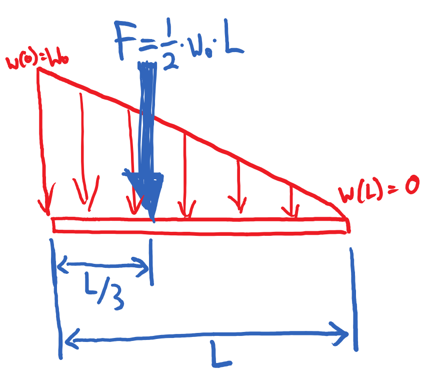
For a triangular loading.

$$\boxed{M} = \int_0^L w_0 \cdot \left(1 - \frac{x}{L}\right) \cdot x \cdot dx = w_0 \cdot \int_0^L \left(x - \frac{x^2}{L}\right) \cdot dx$$

$$= w_0 \cdot \left(\frac{x^2}{2} - \frac{x^3}{3L}\right)\Big|_0^L = w_0 \cdot \left(\frac{L^2}{2} - \frac{L^3}{3L}\right) = \boxed{\frac{w_0 \cdot L^2}{6}}$$

The location of F can then be found:

$$\boxed{\bar{x} = \frac{M}{F} = \frac{\left(\frac{w_0 \cdot L^2}{6}\right)}{\left(\frac{w_0 \cdot L}{2}\right)} = \frac{L}{3}}$$

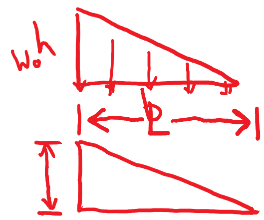


Alternatively, we can use geometric relations:

Since $F = \int_0^L w(x) \cdot dx$ gives the area under $w(x)$,

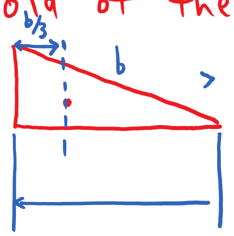
and $w(x)$ is a triangular loading, we can use the formula for the area of a triangle:

Here, $b \to \text{Area}_\Delta$ and $\frac{1}{2} \cdot hb \to w_0$




$$\Rightarrow F = \frac{1}{2} \cdot w_0 \cdot L$$

The location $\bar{x}$ where F acts is through the geometric centroid of the triangle:




So, if $b \to L$, then $\bar{x} = \frac{L}{3}$

$$\boxed{F = \frac{w_0 \cdot L}{2} \text{ acts at } \bar{x} = \frac{L}{3}}$$

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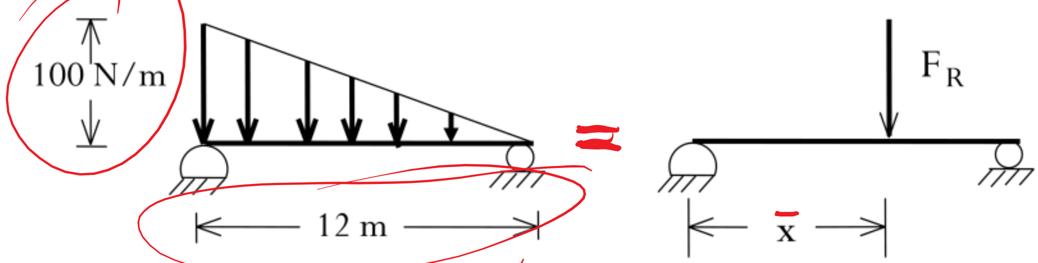
# I>Clicker questions:




$\bar{x} = \frac{12m}{3} = 4m$

1. $F_R$ = ______________

A) 12 N B) 100 N

C) 600 N D) 1200 N

2. $\bar{x}$ = ______________

A) 3 m B) 4 m

C) 6 m D) 8 m

$F_R = \frac{1}{2}(12m)(100\frac{N}{m})$

$= 600 N$